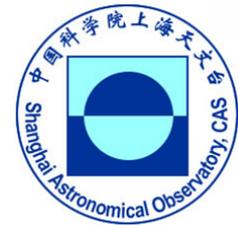


# Modeling of Thermospheric Neutral Density Variations in Response to Geomagnetic Forcing using GRACE Accelerometer Data



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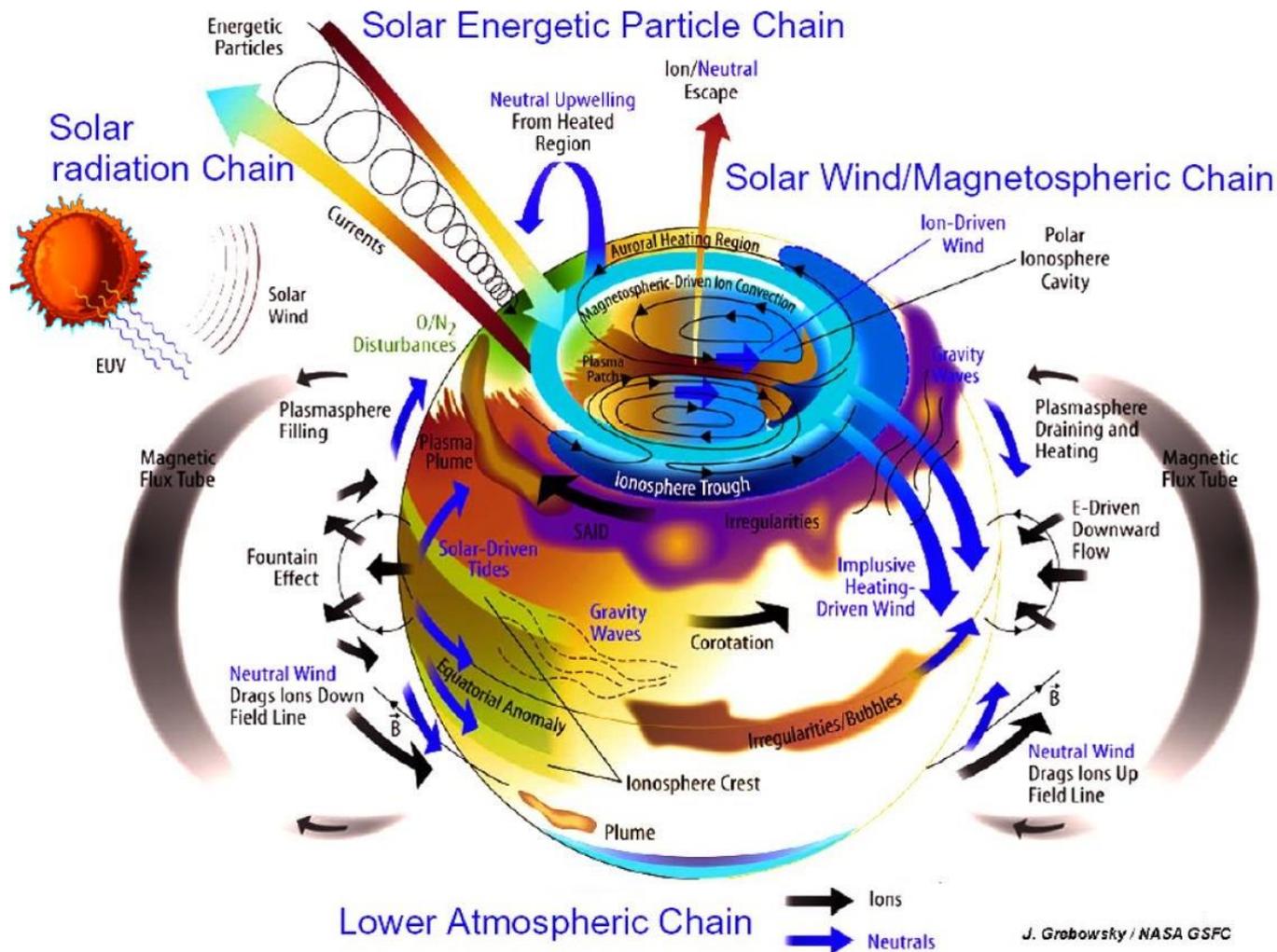
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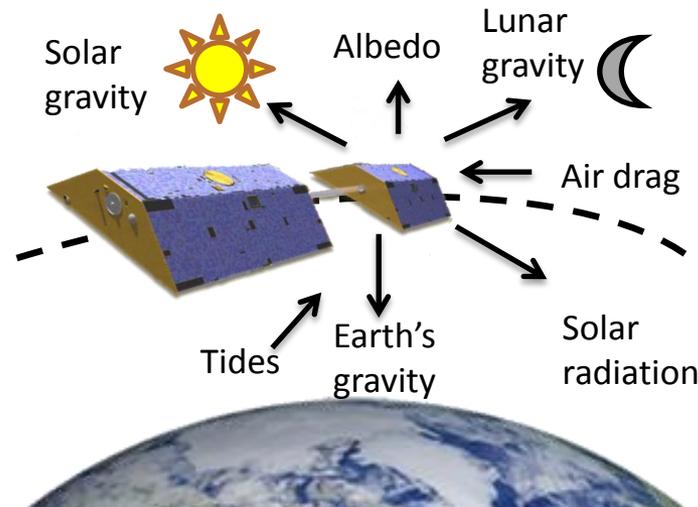
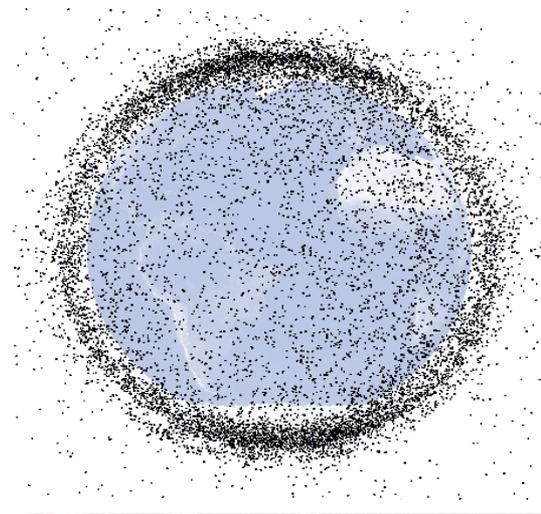
# 1. Introduction & Background

Upper atmosphere processes are not well understood and the current geophysical models are unable to predict the variability as accurately and efficiently required.



# 1. Introduction & Background

Half of the world's active satellites (~1000) and about 20,000 inactive debris operate in LEO, where atmospheric drag produce orbital decay and perturbations.



# 2. Methods & Data processing

## Drag force for density retrieval

- Drag-force formula:

$$F_D = ma_D = \frac{1}{2} CA\rho v_r^2$$

$C$  Drag coefficient (Cook, 1965; Metha et. al, 2013)

$A$  Cross-sectional area

$\rho$  Atmospheric density

$v_r$  Relative velocity of the atmosphere

$m$  Satellite mass

$a_D$  Aerodynamic acceleration

- Normalization to common altitude :

$$\rho(475km) = \rho_{obs}(h) \frac{\rho_{mod}(475km)}{\rho_{mod}(h)}$$

# 2. Methods & Data processing

## Aerodynamic acceleration

Radiation-pressure removal:

### Solar radiation

$$a_{sr} = \sum_{i=1}^{n_p} - \frac{E_{sr} A_i \hat{n}_i \hat{S}_{Sun}^{sat}}{mc} \left[ 2 \left( \frac{c_{rd,i}}{3} + c_{rs,i} \hat{n}_i \hat{S}_{Sun}^{sat} \right) \hat{n}_i + (1 - c_{rs,i}) \hat{S}_{Sun}^{sat} \right]$$

### Earth albedo

$$a_{ea} = \sum_{i=1}^{n_p} \sum_{j=1}^{grid} - \frac{E_{ea,j} A_i \hat{n}_i \hat{S}_j^{sat}}{mc} \left[ 2 \left( \frac{c_{rd,i}}{3} + c_{rs,i} \hat{n}_i \hat{S}_j^{sat} \right) \hat{n}_i + (1 - c_{rs,i}) \hat{S}_j^{sat} \right]$$

$$E_{ea,j} = E_{ea}^R + E_{ea}^{IR}$$

$$E_{ea,j}^R = f_j v_j E_{sr} \frac{A_j (\hat{n}_j \hat{S}_j^{Sun}) (\hat{n}_j \hat{S}_j^{sat}) \sigma_{TOMS}}{\pi |\hat{S}_j^{sat}|^2}$$

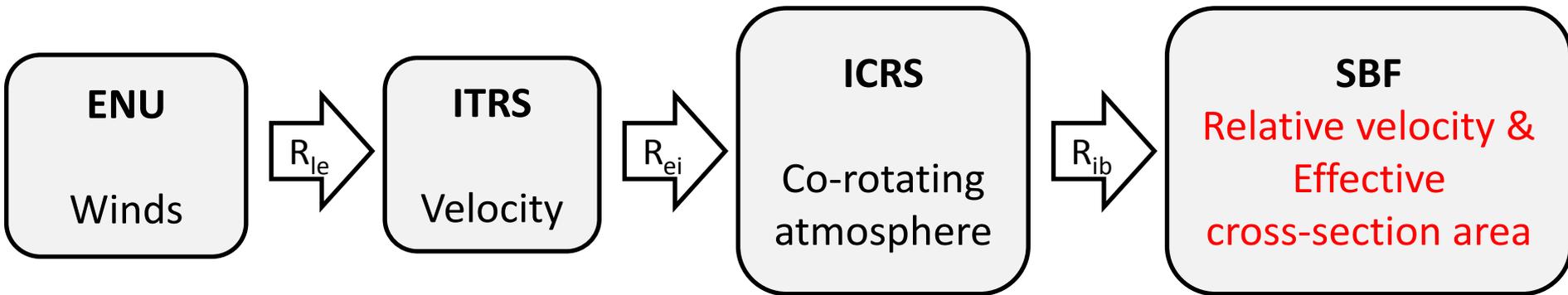
$$E_{ea,j}^{IR} = f_j 239 \left( \frac{1AU}{S_{Sun}^j} \right)^2 e_{IR,j} \frac{A_j \hat{n}_j \hat{S}_j^{sat}}{\pi |\hat{S}_j^{sat}|^2}$$

$$a_D = a_{ng} - a_{sr} - a_{ea}$$

\*Luthcke et al. (1997)

## 2. Methods & Data processing

Reference systems in density retrieval



$R_{ei}$  rotation Earth-fixed to ICRS :

$$\dot{\mathbf{r}}_{ICRS} = [PREC][NUT][ST] \{ [PM] \dot{\mathbf{r}}_{ITRS} + \boldsymbol{\omega}_{\oplus} \times \mathbf{r}_{ITRS} \}$$

$R_{ib}$  rotation ICRS to SBS by using star camera quaternion.

Relative velocity of the atmosphere with respect to the spacecraft

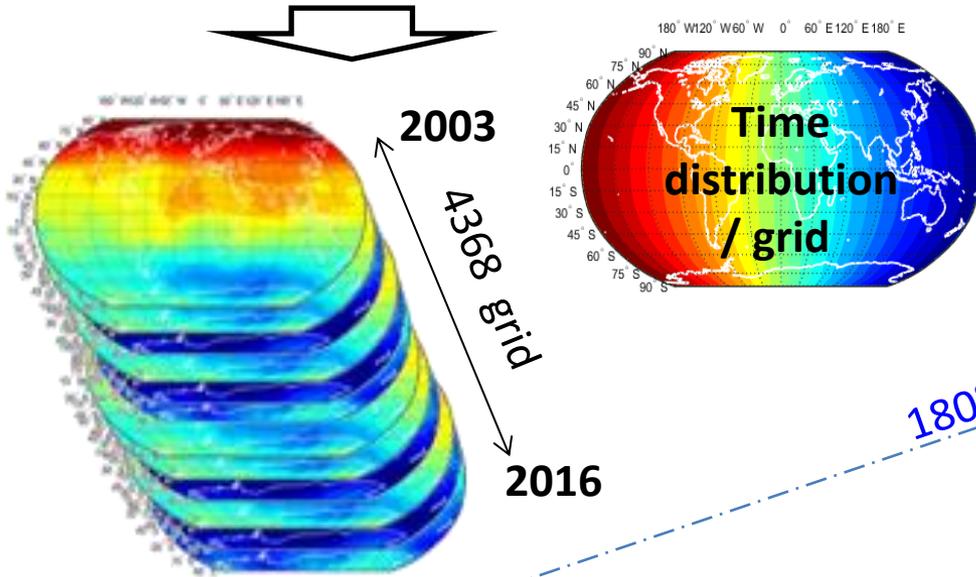
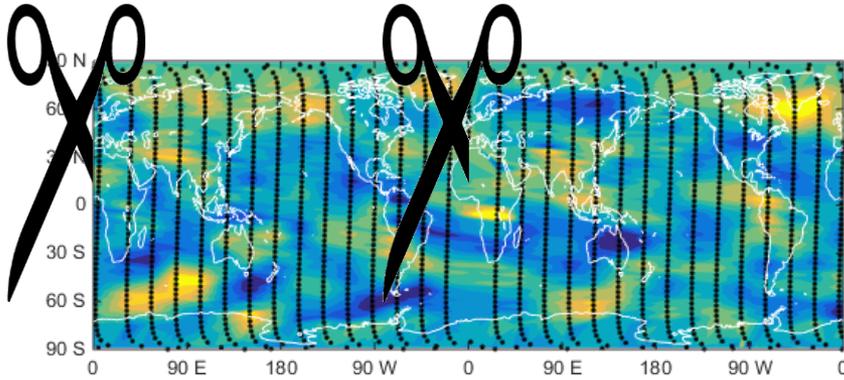
$$\mathbf{v}_{\mathbf{r}} = -\dot{\mathbf{r}} + \mathbf{v}_{\mathbf{r},\mathbf{c}} + \mathbf{v}_{\mathbf{r},\mathbf{w}}$$

Horizontal winds from HWM07 and the co-rotating atmosphere:

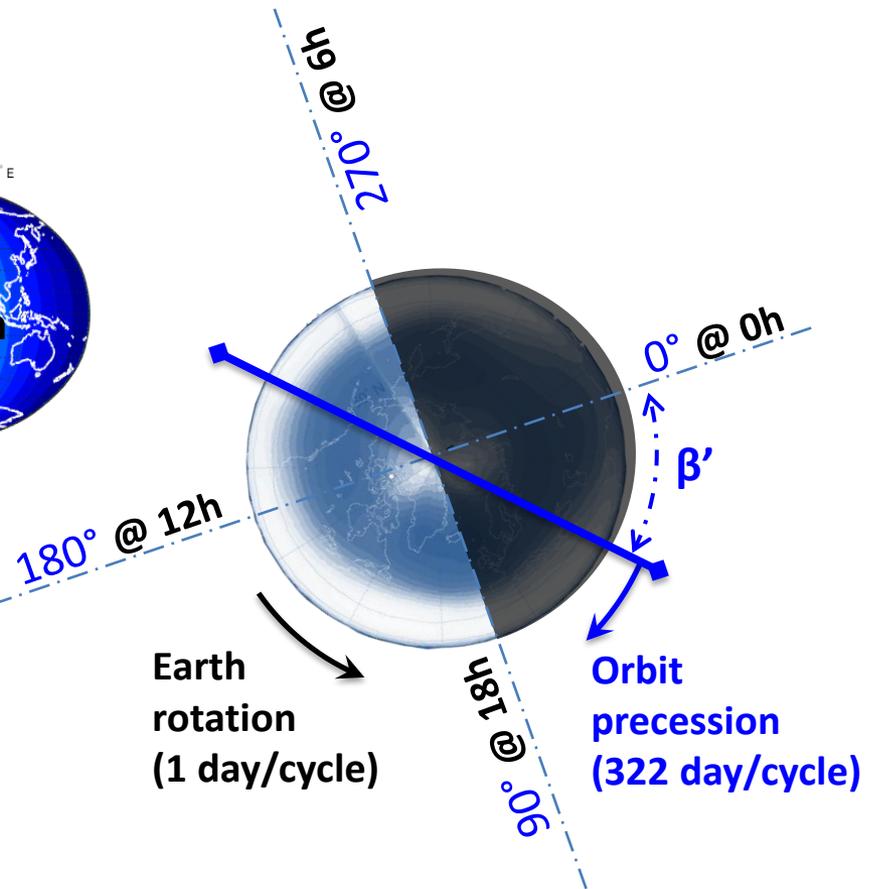
$$\mathbf{v}_{\mathbf{r},\mathbf{c}} = \boldsymbol{\omega}_{\oplus} \times \mathbf{r} = R_{ei}[0, 0, 0.7292115 \cdot 10^{-4} \text{sec}]^T \times \mathbf{r}$$

# 2. Methods & Data processing

- 1<sup>st</sup> Density along orbit
- 2<sup>nd</sup> Data interpolation
- 3<sup>rd</sup> Grid clipping



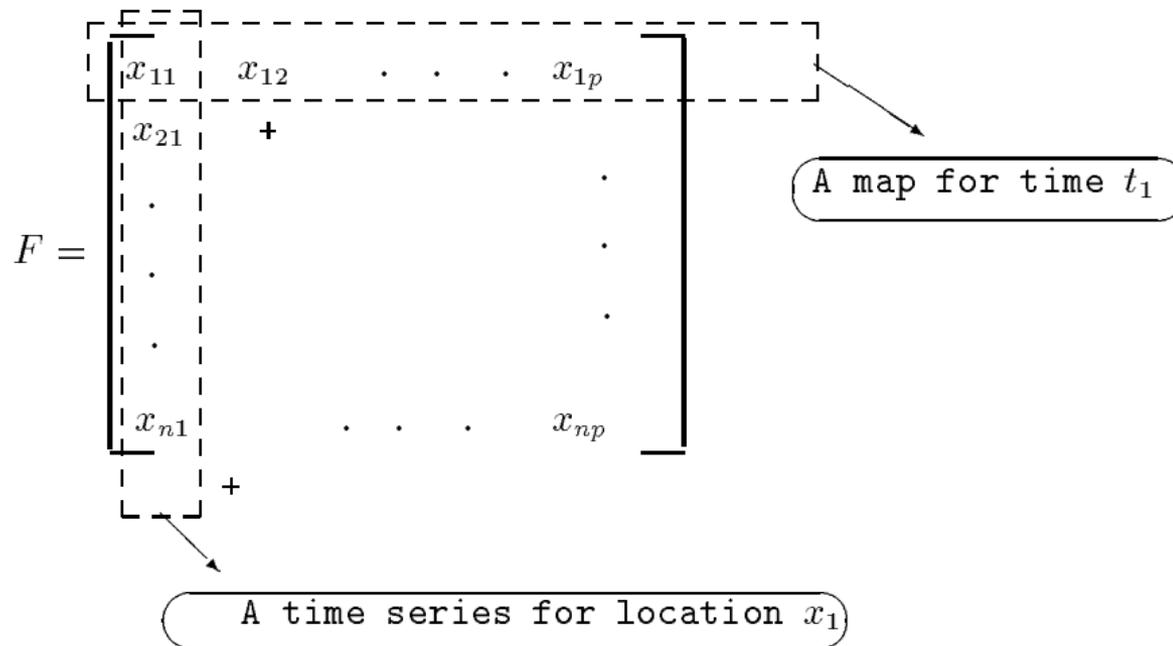
Annual variation  
(365 day)



## 2. Methods & Data processing

### Principal Component Analysis (PCA)

4<sup>th</sup> Arrange each grid in a column.



5<sup>th</sup> Find the covariance matrix.

6<sup>th</sup> Find eigenvalues (time-coefficients) & eigenvectors (maps).

## 2. Methods & Data processing

Parameterization of time-expansion coefficients

**7<sup>th</sup> Normalization to common flux (Muller et al. 2009):**

$$\rho(P10.7 = 110) = \rho \frac{Fa(P10.7 = 110)}{Fa(P10.7)}$$

**8<sup>th</sup> Fourier least-squares fitting:**

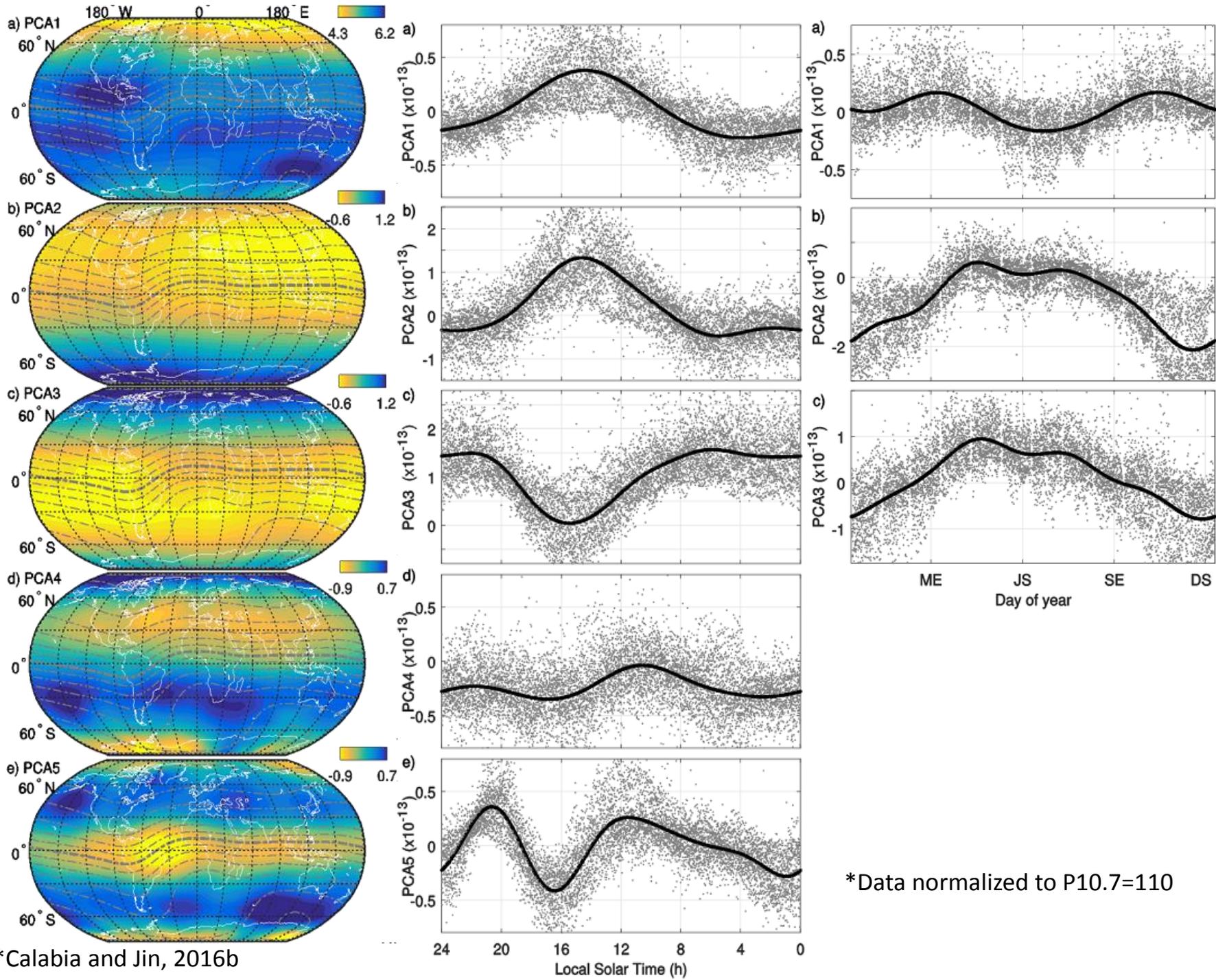
$$\sum_{i=1}^n [a_n \cdot \cos(n \cdot \chi \cdot w) + b_n \cdot \sin(n \cdot \chi \cdot w)]$$

**9<sup>th</sup> Polynomial fitting modulates the amplitude of the sinusoidal function computed in previous step:**

$$G(\chi, P107) = 10^{-15} \cdot 10^a \cdot P107^b \cdot \left( a_0 + \sum_{i=1}^n [a_n \cdot \cos(n \cdot \chi \cdot w) + b_n \cdot \sin(n \cdot \chi \cdot w)] \right)$$

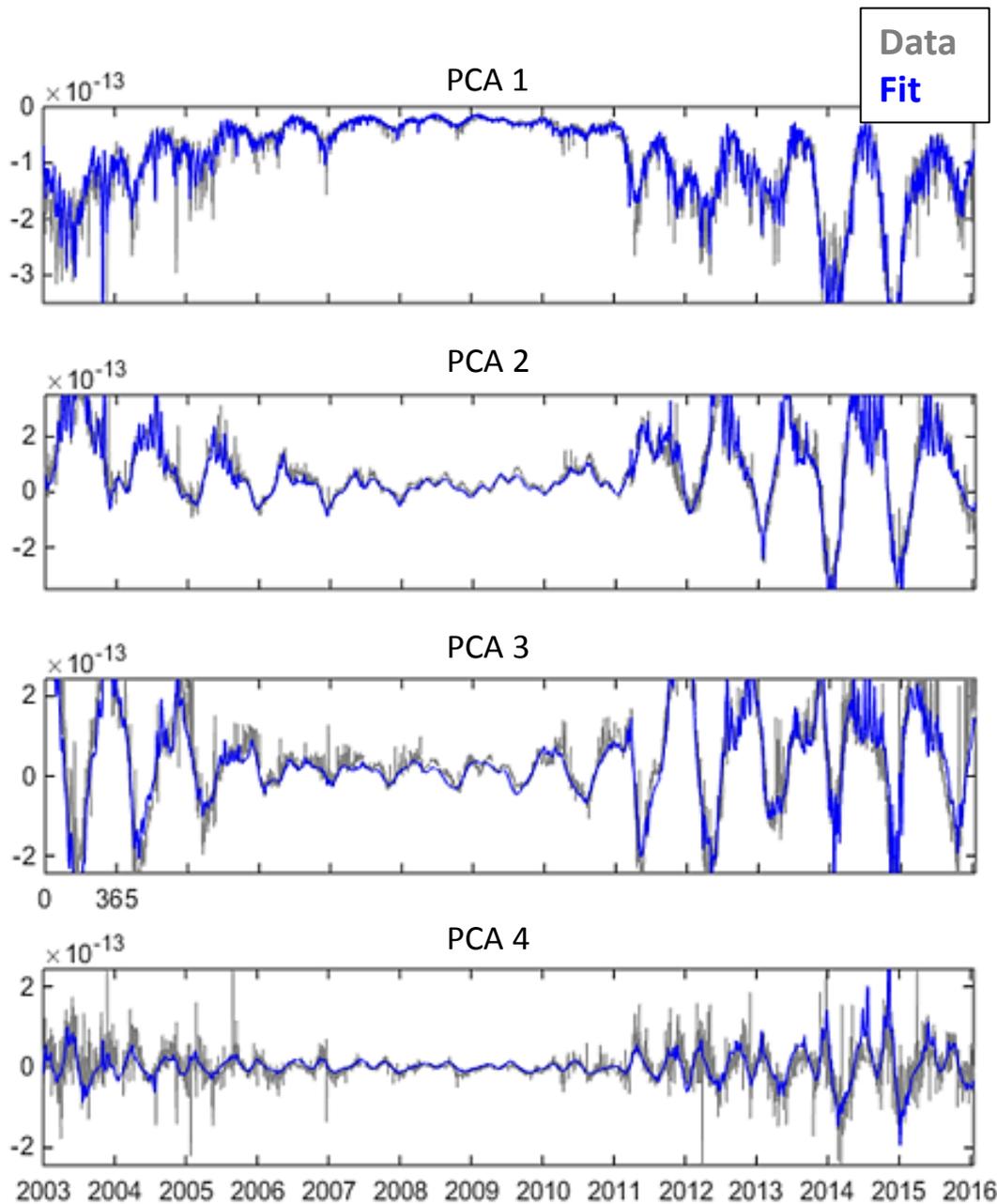
\* a, b, a<sub>0</sub>, a<sub>n</sub>, b<sub>n</sub> and w are the constant and amplitudes, and  $\chi = (\text{doy}, \beta')$ .

# PCA parameterization



# PCA parameterization

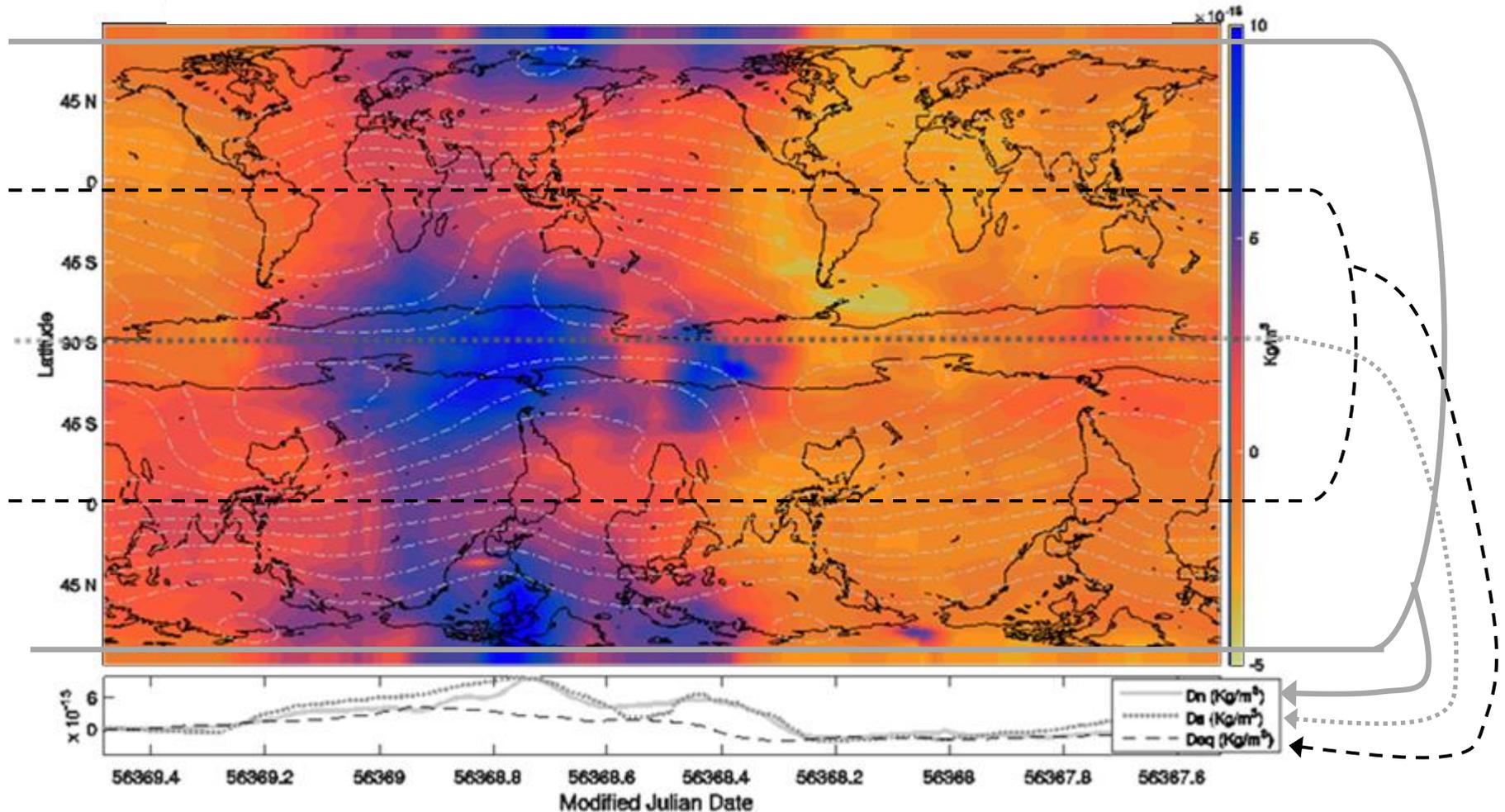
Data vs Fit



| Variance explained | Data - Fit correlation |
|--------------------|------------------------|
| 92 %               | 96 %                   |
| 3.5%               | 93 %                   |
| 3%                 | 90 %                   |
| 1.3 %              | 83 %                   |

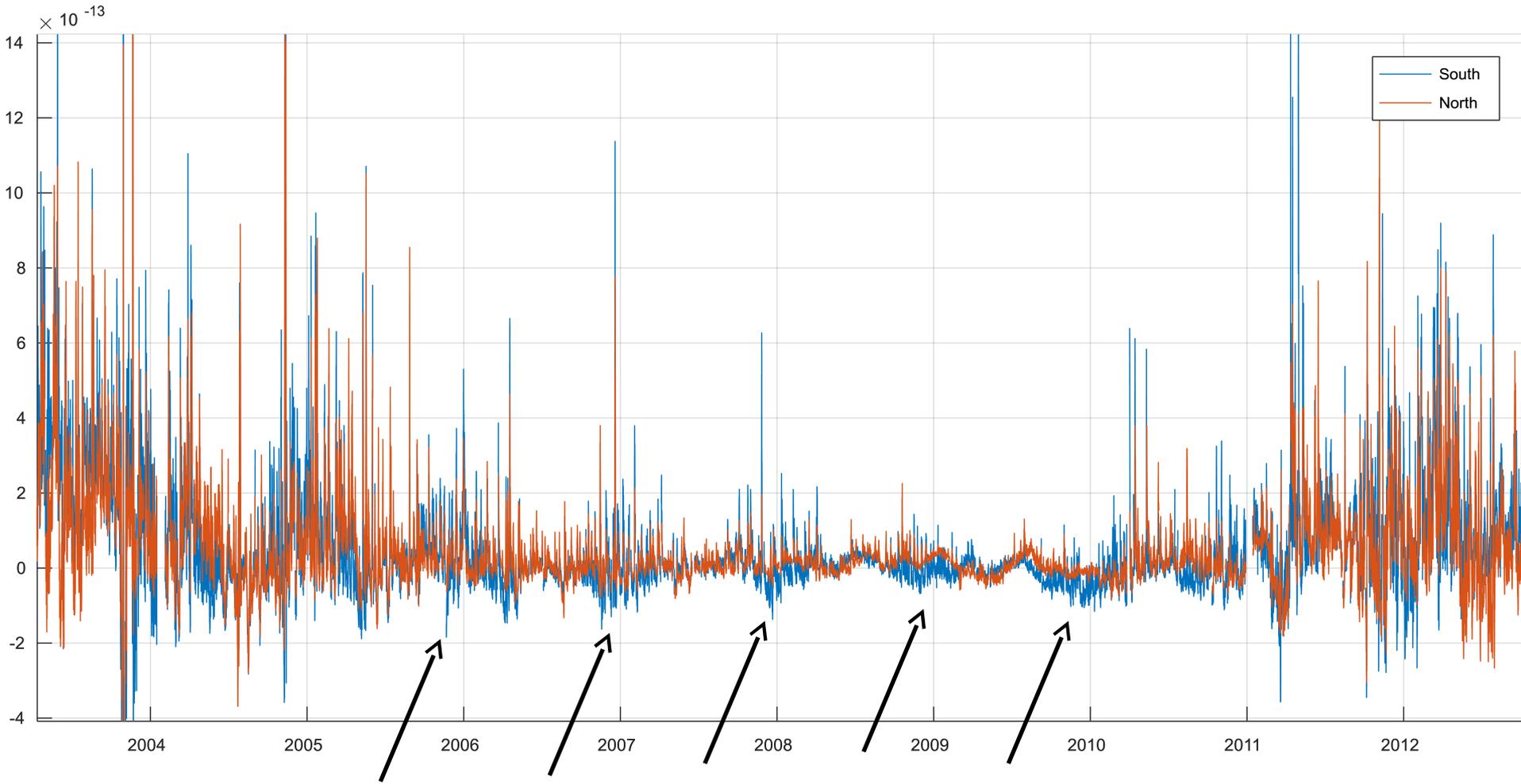
# Extracting profiles of residuals

Northern, Equatorial, and Southern profiles



# 3. Questions & Hypothesis

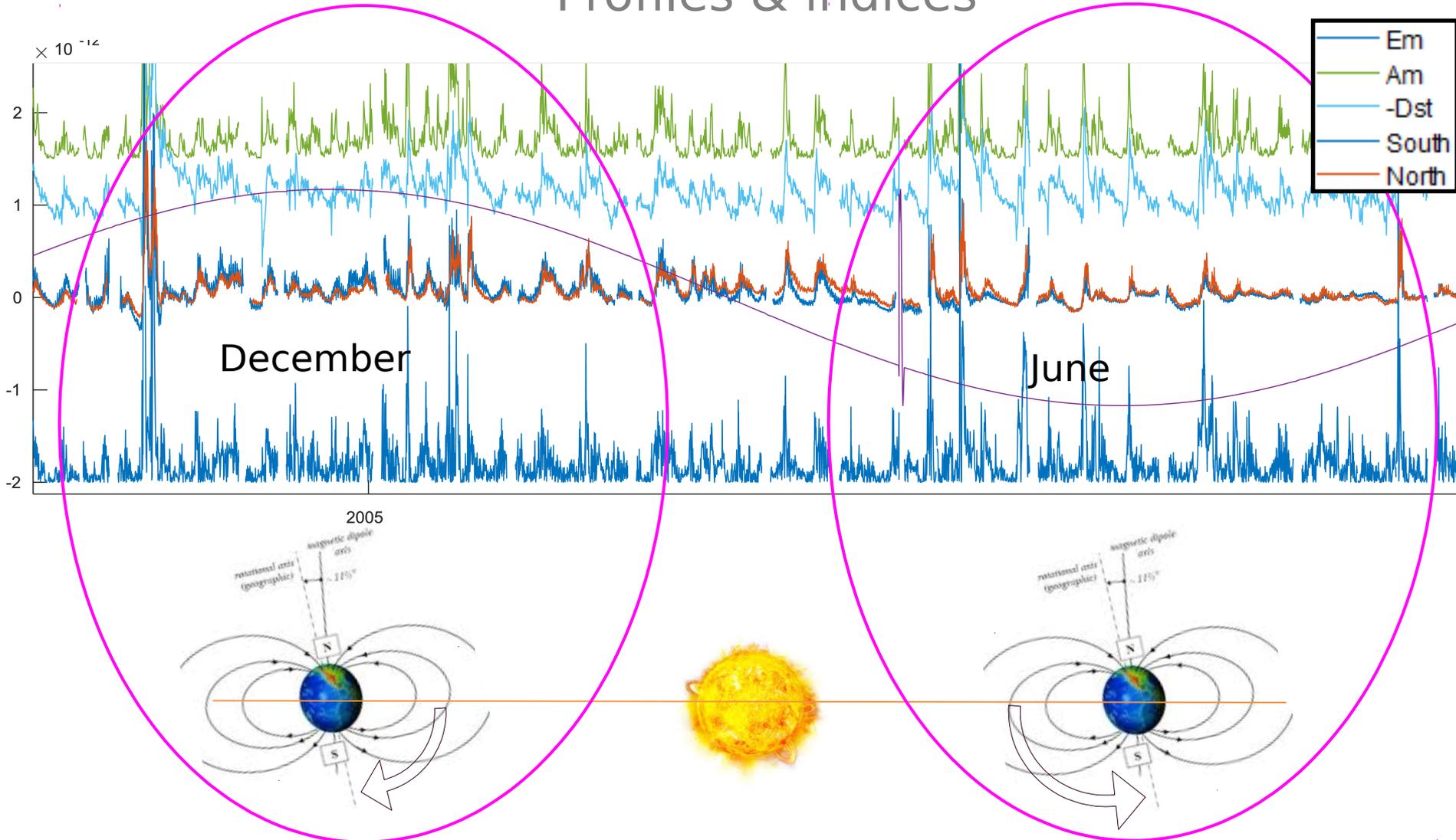
North & South profiles of density residuals ( $\rho_N, \rho_S$ )



**Stronger fluctuations in December in the Southern region !**

# 3. Questions & Hypothesis

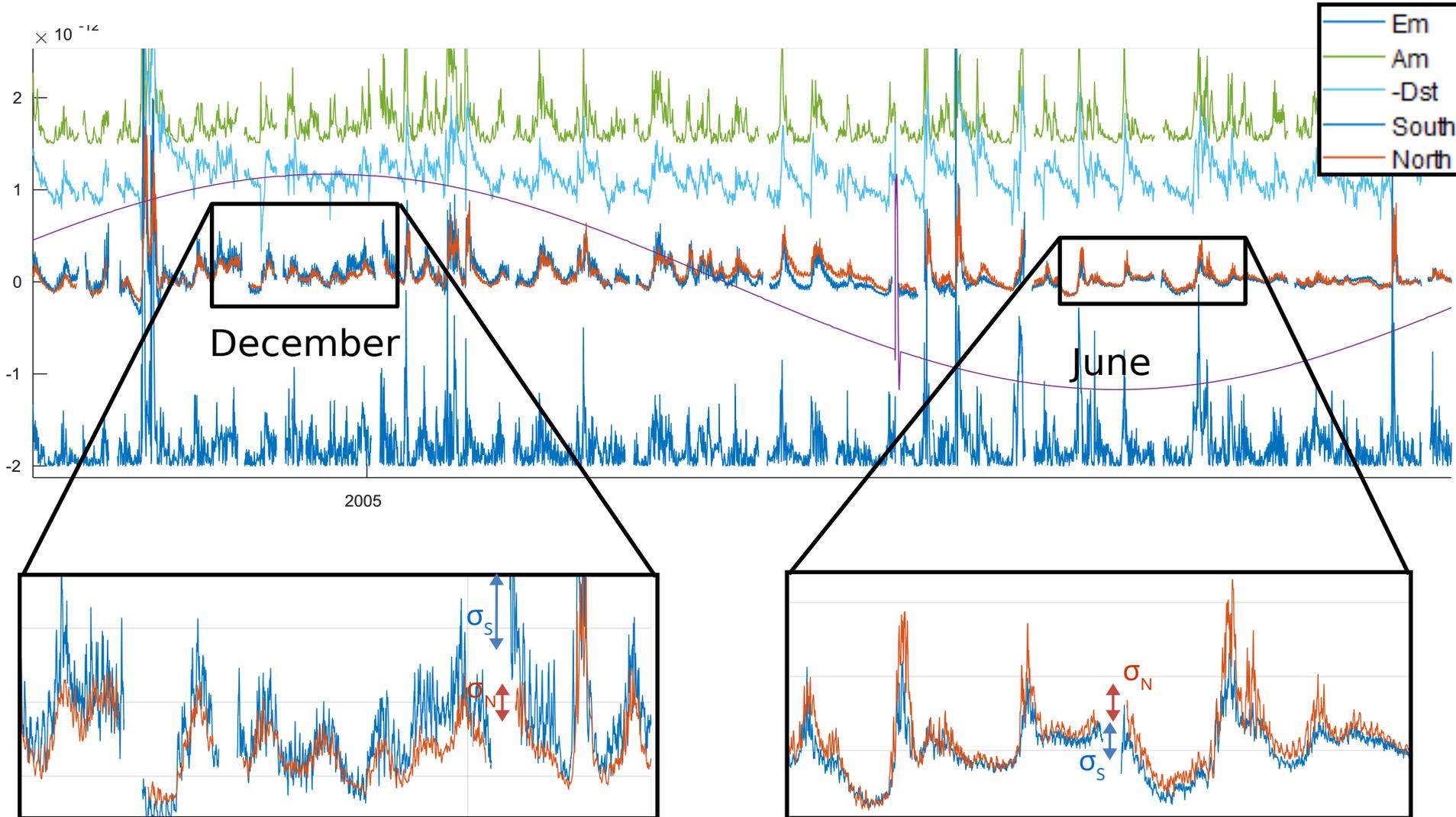
## Profiles & indices



**Density variations attenuate in June solstice (bigger mean magnetic dip angle) !**

# 3. Questions & Hypothesis

## Profiles & indices



**Southern density fluctuations have bigger amplitude ( $\sigma$ ) in December !**

### 3. Questions & Hypothesis

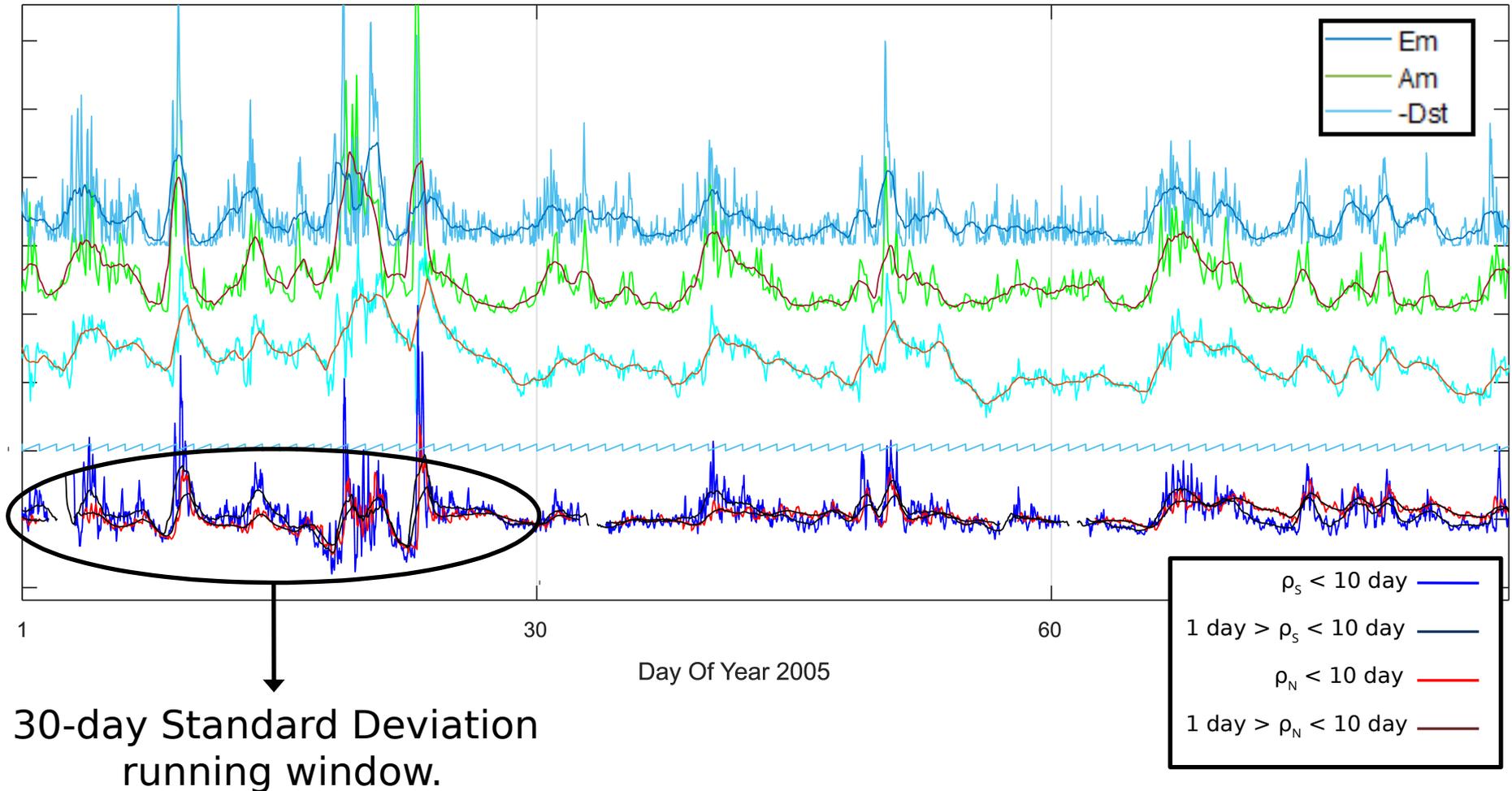
Are these variations a typical storm-time behavior?

A first-principle physical-model (e.g., TIEGCM) would reproduce these variations?

Can this behavior be modeled using the most representative proxies?

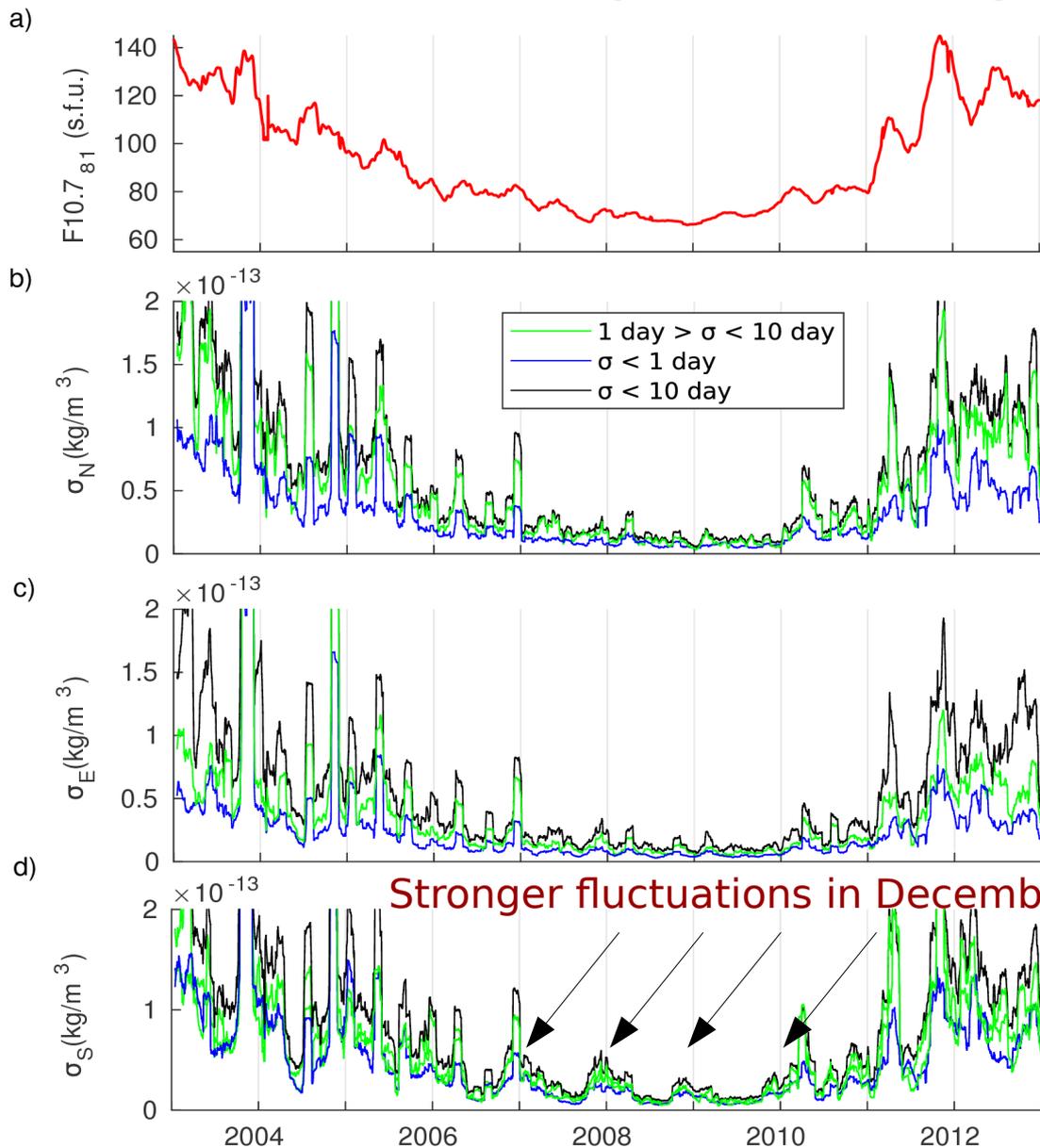
# 4. Analysis

24 h & 30 day mean average running-window



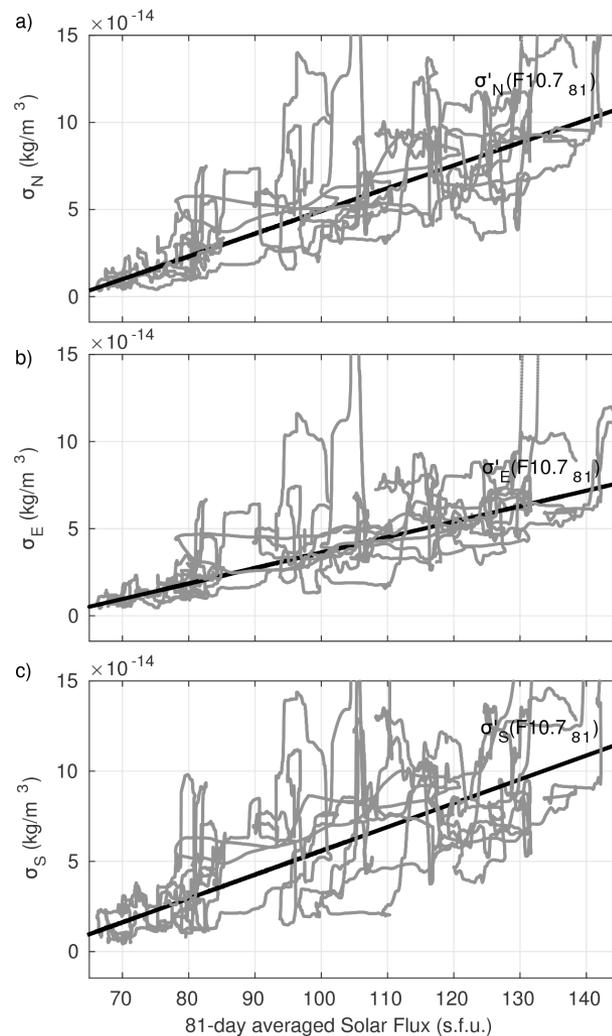
# 5. Results

30-day STD ( $\sigma$ ) running-window  $\propto$  F10.7<sub>81</sub>



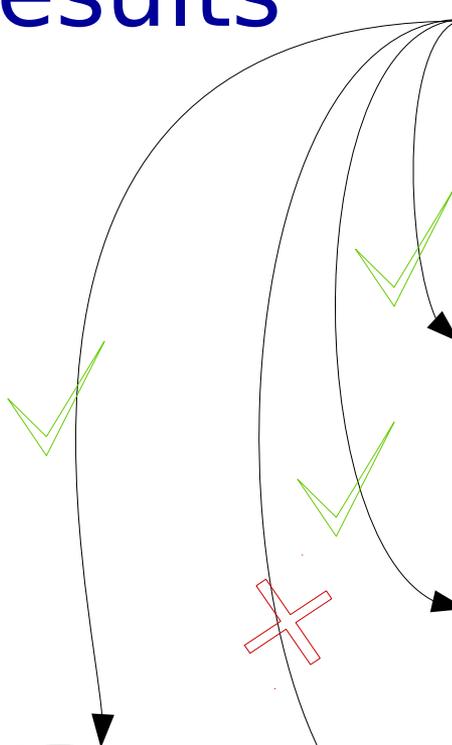
Linear Fitting:

$$\sigma'(F10.7_{81}) = p1 \cdot F10.7_{81} + p2$$



# 5. Results

Agreement with proxies



Fourier Fitting:

$$\sigma_s / \sigma'_s \approx \sigma''_s(\text{doy}) = 1 + a1 * \cos(\text{doy}) + b1 * \sin(\text{doy})$$

Coefficients ( 95% confidence ):

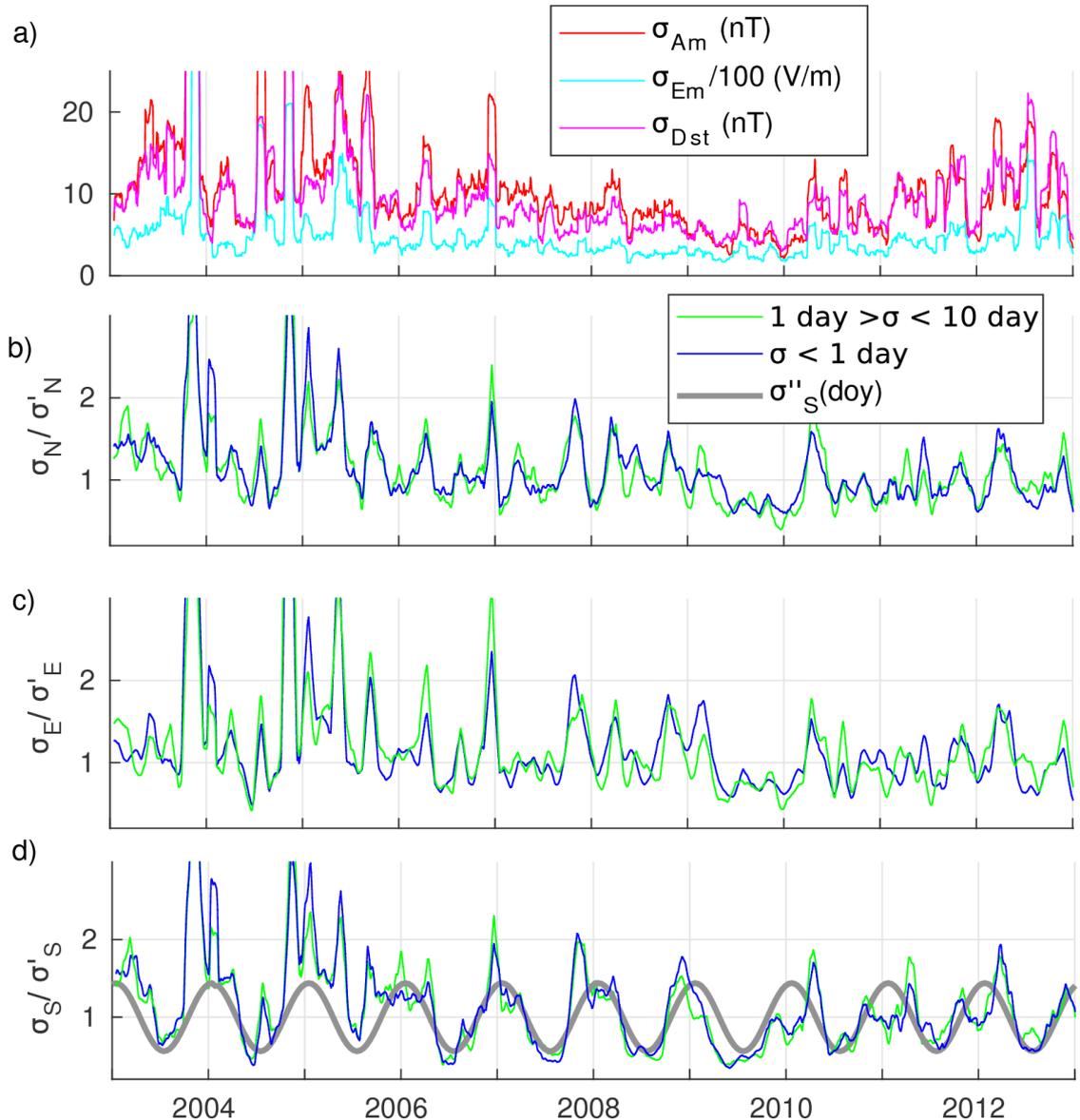
a1 = 0.4268 (0.4256, 0.428)

B1 = 0.1016 (0.1004, 0.1027)

Goodness of fit:

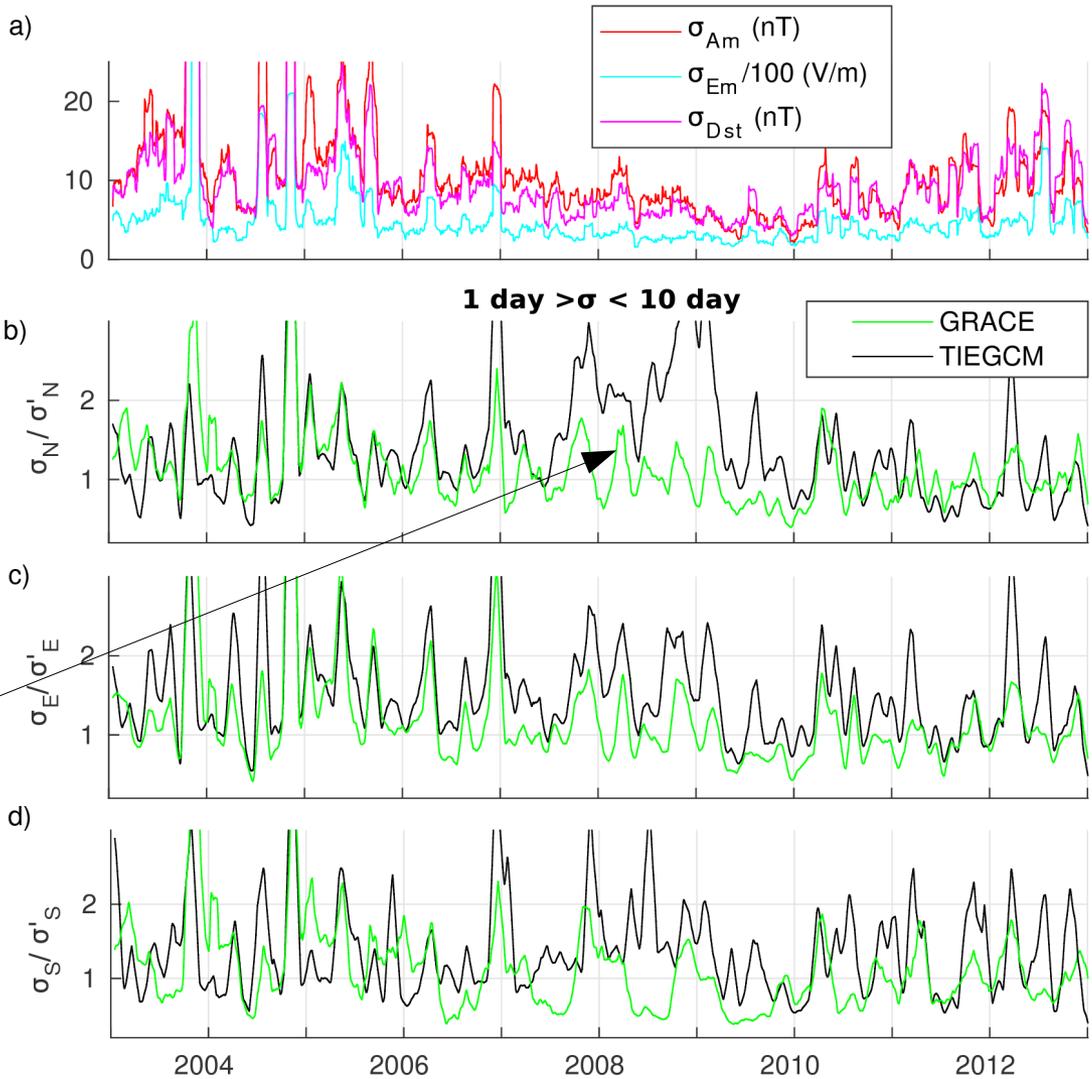
R-square: 0.8

RMSE: 2.62e-01



# 5. Results

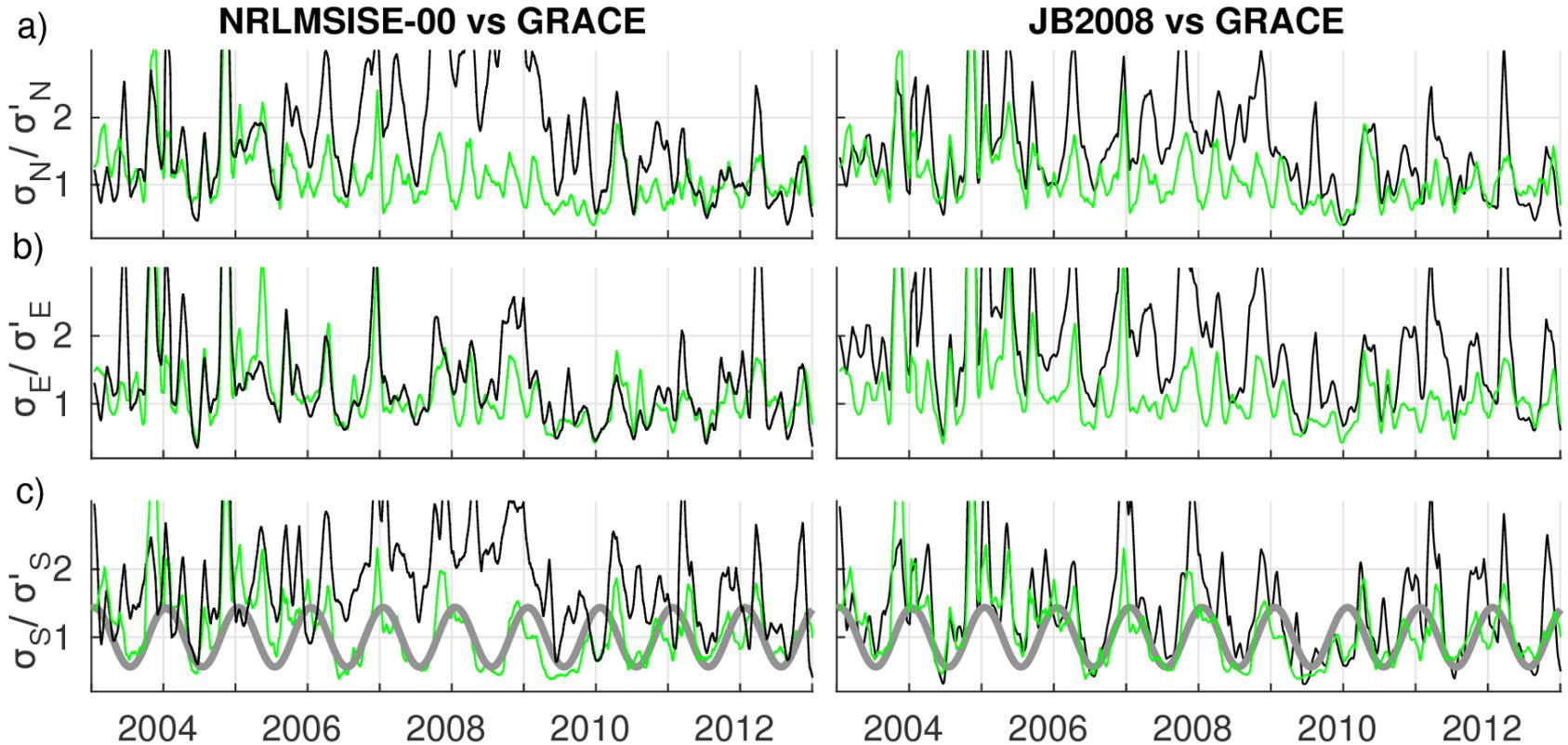
GRACE  
VS  
TIEGCM



TIEGCM shows bigger amplitudes in December in the Northern Polar Cap instead!

# 5. Results

## GRACE vs Empirical models



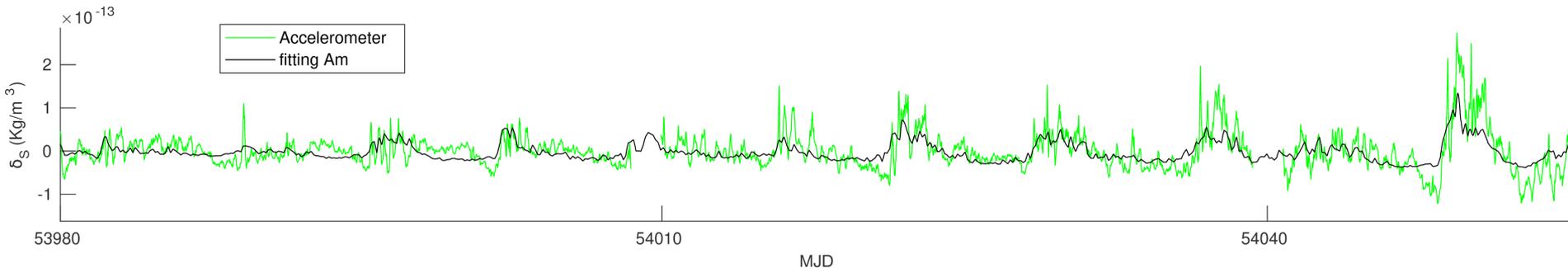
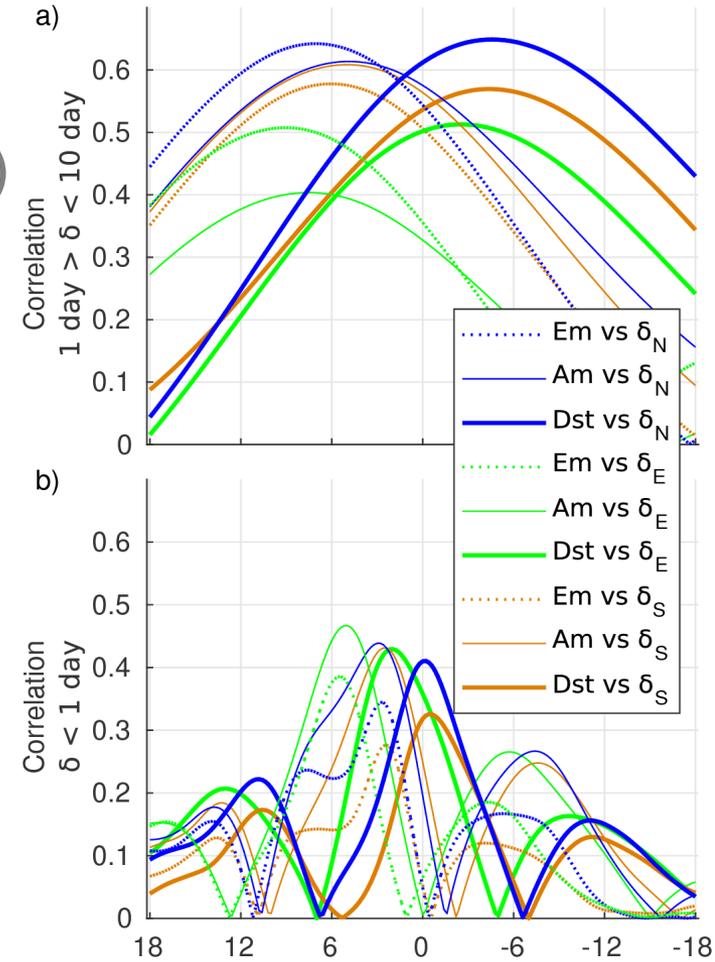
**NRLMSISE00**  
overestimates in low solar  
activity al Polar Caps

**JB2008**  
impressive agreement  
in the South!

# 5. Results

Model in terms of indices.  
Correlation vs Delay (2003-2015)

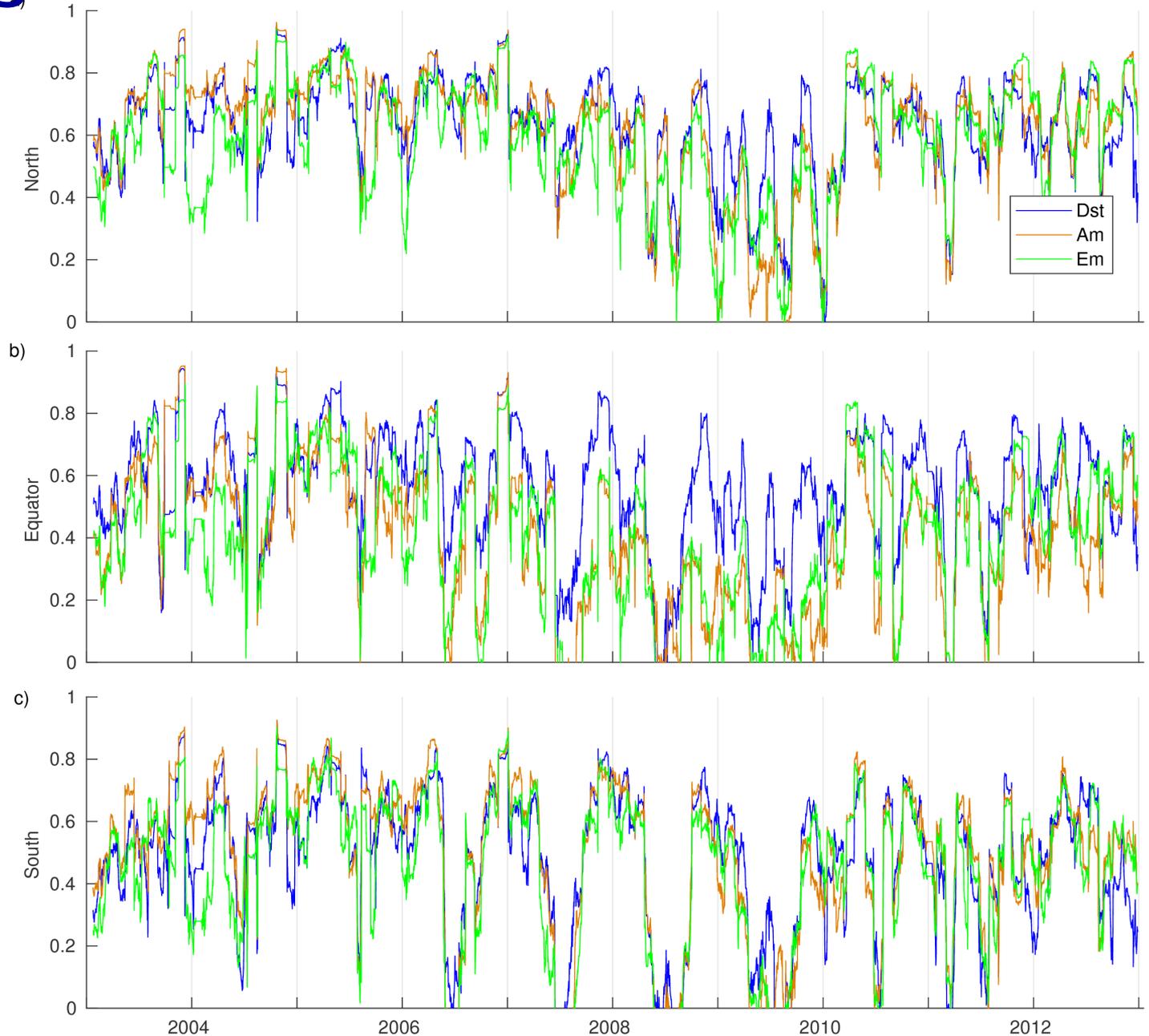
|                           |            |     | R-square | RMSE (kg/m <sup>3</sup> ) | Correlation | Delay (h) |
|---------------------------|------------|-----|----------|---------------------------|-------------|-----------|
| 1 day > $\delta$ < 10 day | $\delta_N$ | Dst | 0.93     | 2.0E-14                   | 0.65        | -4.80     |
|                           |            | Am  | 0.93     | 2.0E-14                   | 0.61        | 4.60      |
|                           |            | Em  | 0.91     | 2.1E-14                   | 0.64        | 6.80      |
|                           | $\delta_E$ | Dst | 0.90     | 1.9E-14                   | 0.51        | -2.80     |
|                           |            | Am  | 0.87     | 2.1E-14                   | 0.36        | 7.40      |
|                           |            | Em  | 0.87     | 2.1E-14                   | 0.50        | 8.80      |
|                           | $\delta_S$ | Dst | 0.90     | 2.6E-14                   | 0.56        | -4.60     |
|                           |            | Am  | 0.90     | 2.6E-14                   | 0.62        | 4.80      |
|                           |            | Em  | 0.90     | 2.7E-14                   | 0.56        | 5.80      |
| $\delta$ < 1 day          | $\delta_N$ | Dst | 0.89     | 1.7E-14                   | 0.45        | -0.40     |
|                           |            | Am  | 0.89     | 1.6E-14                   | 0.44        | 2.80      |
|                           |            | Em  | 0.89     | 1.6E-14                   | 0.38        | 2.60      |
|                           | $\delta_E$ | Dst | 0.91     | 1.3E-14                   | 0.43        | 1.80      |
|                           |            | Am  | 0.91     | 1.2E-14                   | 0.50        | 4.80      |
|                           |            | Em  | 0.91     | 1.2E-14                   | 0.44        | 5.20      |
|                           | $\delta_S$ | Dst | 0.89     | 2.5E-14                   | 0.38        | -0.60     |
|                           |            | Am  | 0.90     | 2.4E-14                   | 0.42        | 2.40      |
|                           |            | Em  | 0.90     | 2.4E-14                   | 0.32        | 2.20      |



# 5. Results

Fitting  
VS  
GRACE

Correlation  
at 30-day  
Running  
window



# 6. Conclusions

- Neutral density variations due to geomagnetic forcing are strongly dependent on solar flux and dipole-tilt angle variations.
- Southern density variations have bigger amplitude in December, when the southern dipole is oriented to the Sun (good agreement with JB2008). TIEGCM show bigger amplitudes in December in the North instead.
- *Dst* index (variations  $<10$  day) shows worse delay for prediction, while *Am* and *Em* show best suitability for prediction.

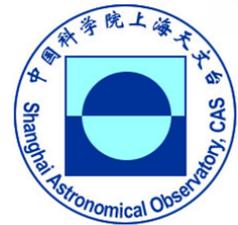
# AGU FALL MEETING

New Orleans 11-15 Dec. 2017

## Thank you!



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